The Trading Game

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My first two papers had generating some alpha for subject line: "***Alpha Power: Adding More Alpha to Portfolio Return***" and "***A Jensen Modified Sharpe Ratio to Improve Portfolio Performance***". Follow-up notes where also made available here.

In both papers and notes, elaborate equations were provided to make the point that not only can some alpha be generated, but that it was relatively easy to do so. It was shown that the ability to pick stocks with above average performance counts, but that it was the ability to continue to reinvest part of the profits, in the same manner that generated the profits in the first place, that really mattered.

This paper goes almost full circle as for conclusion it recommends a Warren Buffett style of trading. In a sense this is my tribute to the way he plays the game. It presents in mathematical form what he has done all along. In my view, Mr. Buffett is a star, and hopefully, not alone.

At first, this was not the direction I had undertaken; I was looking for something else. But as my research evolved, it took a direction of its own. It still maintains and reemphasizes what was presented in the previous papers and notes meaning that the proposed trading strategy can indeed outperform the Buy & Hold by using simple reinvestment procedures and that was sufficient to generate some alpha.

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The Game:

It was shown in previous papers that it might be necessary, if not at least preferable, to look at the portfolio management problem from a total solution perspective. The game is not a single period game. It should be played with specific long term and reachable objectives. Solving the problem for a single period is a delusion unless this single period under consideration is the whole time frame; in which case it becomes another representation of the Buy & Hold investment strategy which should in itself be considered as the trivial solution in the sense that practically no work or time need be devoted to finding performance enhancers. The Buy & Hold is simply a stock picking strategy, and using an over-diversification approach; you could allocate all your capital to a group of index funds, thereby achieve a long term performance close to the overall average market return.

The pay-off matrix notation as used by Schachermayer was seen as a mathematical simplification of the whole trading game. Only two variables required to describe any type of trading strategy: a quantity held over a time interval subject to price variations.

$$(H \cdot S)_t = \sum_{j=1}^{t} (H_j, \Delta S_j) \qquad (1)$$

where $\Delta S = P_t - P_{t-1}$ stand for the price variation and $S_j(t)$ stand for the j th stock of d ($1 \leq j \leq d$) spanning the investment horizon $t \in [0,T]$; and $H_j(\cdot)$ the quantity of shares held in a specific stock over time. Equation (1) represents the cumulative total profit or loss generated by the trading strategy.

$$(H \cdot S)_t = \sum_{n,j=1}^{n,t} (H_j, (P_{j_t} - P_{j_{t-1}}))$$

While trying to solve this position sizing matrix, it was noted that not much could be done to change future prices as they are to be what they will be; but that much could be done on the inventory side of the equation and on the stock selection process itself. It was first proposed to have an enhanced holding function seeking to out-perform the Buy & Hold strategy which was easily formulated as:

$$(H \cdot S)_T = \sum (h_0^j \mathbf{I}, \Delta S_j)_T \qquad \text{Buy \& Hold strategy}$$

where h_0^j was the initial stock purchase in each of the d stocks in the portfolio, $\mathbf{I}$ a matrix composed entirely of ones, and T the terminal time horizon for the investment period. Using H^+ to represent an improved inventory process over the Buy & Hold strategy would only require a higher valued enhanced holding matrix such that: $H^+ > H$. This enhanced holding matrix outperforms the Buy & Hold matrix and can be stated as follows:

$$(H^+ \cdot S)_T > (H \cdot S)_T \quad \text{or alternatively} \quad (H^+ \cdot S)_T - (H \cdot S)_T > 0$$

which tries to express the positive benefit extracted using an enhanced holding matrix. This enhanced holding matrix H^+ will out-perform what may be considered as the most probable outcome (the average secular market return) provided that it had an increasing scaling function over time, as in:

$$(H^+ \cdot S)_T = (k^j(t)H, \Delta S)_T = (k^j(t)h_0^j \mathbf{I}, \Delta S_j)_T \quad \text{with } k^j(t) > 1$$

where $k(\cdot)$ can be a scalar, a linear or a time increasing function matrix which could be designed based on pre-determined long term trading objectives.

This enhanced pay-off matrix $(H^+ \cdot S)_T$ represents a position sizing problem that spans over the entire investment horizon. It is therefore by selecting appropriate stock trading windows from the available stock tradable universe, and all the while, determining suitable bet size that one should seek to achieve over-performance. Designing a trading strategy which has for main objective to increase inventory in time should be relatively simple, or should it?

Since the inventory on hand in each stock is closely related to portfolio allocation and risk management, previous discussions led to find which decision distribution process would improve overall performance. And in turn, this led to the necessity of having a decision surrogate; a process that would decide not only which stock should be traded, but also which quantity should be traded at any given time. The same process consequently would determine the holding period within a tradable window for any one stock in the portfolio as well as the size of each bet. The inventory on hand, as presented in the previous section, would respond to the following equation:

$$QB^j(t) = IB_0^j + \sum_{t=1}^{T} D^j(t) \cdot TM^j(t) \cdot TB_t^j(t) \quad \text{for } 1 \le j \le d \text{ , } t \in [0,T] \qquad (2)$$

The quantity bought over time $QB(\cdot)$ would be the result of the initial quantity purchased IB_0^j to which would be added ongoing trades. The decision surrogate $D(\cdot)$ is used to determine if a trading window is open or not. The trade modifier $TM(\cdot)$ has for task to increase or decrease the trade basis $TB(\cdot)$ serving as a trading unit. Many factors could and can alter the decision process and weigh on trade allocation during the life of the trading system (see previous section: “The Pay-Off Matrix” for a more detailed view).

The Market Pay-Off Matrix:

As a whole, the market can only offer what it has to offer; in the sense that holding all stocks over any time horizon is equivalent to holding the market itself. And the return on this market portfolio, unsurprisingly, would be the average market return:

$$(H_M \cdot S_M)_t = \sum_{j=1,t=0}^{M,T} (H_j, \Delta S_j) = \sum_{j=1,t=0}^{M,T} (h_0^j \mathbf{I}, \Delta S_j) \quad t \in [0,T],\ 1 \le j \le M$$

where M includes all stocks in the market universe. Picking a subset of the market universe is akin to selecting n stocks out of the M stocks available. And as n increases, the average portfolio price variation will tend to mimic the average market price variation.

$$\frac{\frac{1}{n}\sum_i^n dP_i^n(t)}{\frac{1}{m}\sum_i^M dP_i^M(t)} \rightarrow 1 \quad as \quad n \rightarrow M \qquad \text{most surely}$$

Therefore, the more one diversifies his/her holdings, the more the portfolio's average price will tend to the average market price. Taking 30 stocks or more is often considered sufficient to have the above equation exceed 0.95; meaning that, long term, the average portfolio price will be within 5% of the average market price. As the number of stocks increases, the above ratio will tend asymptotically to 1.00 most surely.

To improve the pay-off matrix from the price variation side of the equation, it is required to select stocks that can out-perform the market or alternatively select better trading windows.

$$(H \cdot S^+)_T > (H \cdot S)_T \qquad for\ S^+ > S,$$

where S^+ represent a subset of stock outperforming the market; implying that better stock picking can indeed increase terminal wealth. In fact, ultimately, one could select a single stock - the atlas stock – the one that will outperform all other stocks to obtain the highest possible return. In hindsight, this is always very easy. However selecting today, tomorrow's atlas stock is much much more difficult. Nonetheless, even picking the atlas stock would still not be the "optimal" portfolio as even this can be greatly improved upon with the proposed methodology.

Any market timing ability could also improve overall return by selecting appropriate trading windows on the various stocks in the portfolio. This time slicing of trades has its own pitfalls: one of which is the need to compensate for market under-exposure. If you are in the market 10% of the time, the trades done need to provide higher returns just to compensate for the non-exposure (the times you are not in the market). And even if you provide this higher return, it does not necessarily mean that it will beat, long term, the Buy & Hold strategy. Also, one should consider that time slicing of trades might be fine for smaller portfolios however, when the portfolio gets much larger flipping it daily can be a daunting task.

$$(H \cdot S)_T > (H^- \cdot S)_T \qquad \text{for } H > H^-$$

Under-exposure will generally result in a reduced holding function which will translate into a lower performing portfolio. You need market exposure to generate profits and good market timing to compensate for under-exposure.

Should your attempts at market timing be unsuccessful, with or without under-exposure, it would still translate into a lower performance than the Buy & Hold strategy since this would have been the same as selecting under-performing stocks.

$$(H \cdot S)_T > (H \cdot S^-)_T \qquad \text{for } S > S^-$$

Considering all of the above, an enhanced pay-off matrix should try to make the best of both worlds; trying to pick the best trading stocks possible all the while trying to increase the inventory as time goes by. This would result in:

$$(H^+ \cdot S^+)_T > (H \cdot S)_T \qquad \text{with } H^+ > H \text{ and } S^+ > S$$

where the enhanced holding matrix is operating over an expected out-performing stock selection (your best long term stock forecasts for example).

One could also elect repeated shorter time intervals, this way taking advantage of price volatility. Taking 10% return over a single month, week or day produces the same amount as taking 10% over the year (about 75% of stocks vary by more than 25% - from peak to trough - in any one year, and of those about 25% vary by more than 75% in any single year). Thereby, there is a strong case for short term trading and market timing leading to improved portfolio performance.

This all leads to a very basic question: how should anyone trade? What is the optimal solution? There is no single answer to the question as it will depend on too many factors to be applicable to everyone across the board; especially when dealing with an uncertain future. Trading a small, medium, large or very very large portfolio brings its own set of constraints and limitations.

Time Slicing:

Trading more often, getting in and out of trades over the investment horizon is done under the premise that one can time the market or at least the trade being undertaken. But often, one is kind of obligated to terminate a trade in order to limit losses on a stock moving in the wrong direction and this no matter what fundamental or technical analysis might say.

When looking at a stock price one could subdivide the time series in as many segments as desired. Using, for example, a 1000-week price series, one could opt to buy every Monday at the open and sell every Friday at the close. The result would be 999 trades where the accumulated profits would almost surely approach the profits generated by Buy & Hold strategy. There tends to be no benefit in time slicing trades in this manner. Deciding to trade every other week, or every other month, could only result in under exposure and under performance with higher trading cost than the Buy & Hold scenario. Using indicators to time short term trading windows would have to generate more profits than the lost trading opportunities for under-exposure as well as enough to cover any executed stop loss over the investment period. It is not easy to beat the Buy & Hold strategy using conventional techniques.

What is required is selecting trading windows with favorable outcomes from the available tradable window set. This would require introducing some sort of predictive component to the process or a trend following method with a positive expectancy.

Even time slicing of trades has to follow the SDE equations as backdrop; in the sense that these equations still prevail that you time slice your trades or not. We could rewrite the incremental change in price as:

$$\frac{dS}{S} = \mu\, dt + \sigma\, dW$$

and the holding matrix composed of a drift and a stochastic component as:

$$(H \cdot S)_T = \sum (H, \mu\, Sdt + \sigma\, SdW)_T$$

$$(H \cdot S)_T = \sum (H, \mu\, Sdt)_T + \sum (H, \sigma\, SdW)_T$$

The holding function is composed of one part benefiting from the long term drift while the other is subject to random fluctuations. Notice that the Buy & Hold strategy is still expressed as before:

$$(H \cdot S)_T = \sum (h_0^j \mathbf{I}, \Delta S_j)_T \qquad \text{Buy \& Hold strategy}$$

The expected value of the holding matrix would be:

$$\mathbf{E}(H \cdot S) = \mathbf{E}\sum (H, \mu\, Sdt) + \mathbf{E}\sum (H, \sigma\, SdW)$$

The random component would have for expected value 0 as $\mathbf{E}\sum (H, \sigma\, SdW) \to 0 \quad a.s.$. That this random component be scaled by any time dependent function would result in an expected value of 0 as it still represents the error term of a linear regression. And furthermore, for any portfolio the sum of these random components, which tend to zero, would also tend to zero.

On the other hand, positively scaling the drift component would result in improved performance. First, the expect payoff comes from the drift component of the equation:

$$\mathbf{E}(H \cdot S) \to \mathbf{E}\sum (H, \mu\, Sdt) \qquad \text{since} \quad \mathbf{E}\sum (H, \sigma\, SdW) \to 0 \quad a.s.$$

and it is by providing an enhanced holding matrix that one can improve performance:

$$\mathbf{E}(H^+ \cdot S) \to \mathbf{E}\sum ((\mu\,(1+\tau)^t H), \mu\, Sdt)$$

where the enhanced holding function on the drift component is scaled by an exponential function of the drift itself. In fact, it becomes a sufficient condition to reinvest part of the excess equity buildup to improve performance beyond the Buy & Hold strategy.

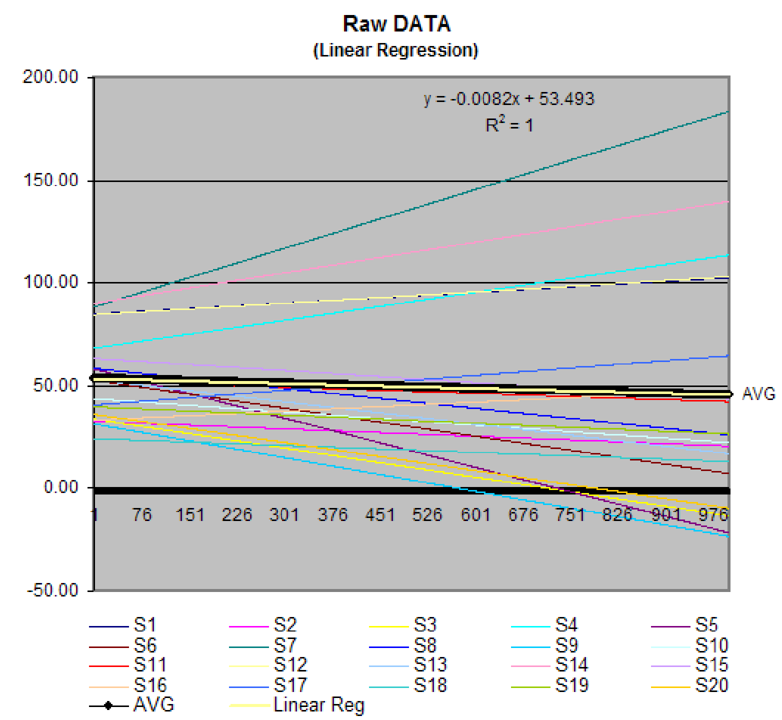
Removing the random component of the equation should be view as a no impact solution since; after all, its expected value is zero. And doing so also puts aside the need to determine the best way to treat exploitable trading strategies dealing with quasi-random data. Since $\mathbf{E}\sum (H, \sigma S dW) \rightarrow 0 \quad a.s.$ and since over-diversification will assure that the portfolio performance $\mu_p(\cdot)$ will tend to the market average return $\mu_M(\cdot)$; it should come as no surprise that not much would be lost by removing the quasi-random nature of price movements from the SDE equation.

This is like sidestepping one of the biggest problems of portfolio management: developing a usable trading strategy that can profit from the random component of the performance equation. Ignoring what amounts to the greatest impact on price movement might sound illogical until you realize that the expected value of trading on the random component itself tends to zero. It is like detrending all price series and then trying to profit from the random fluctuations which have for ultimate expectancy zero.

> *I was faced with a problem where I had a hard time putting it in mathematical terms. What I wanted was to express the probabilistic paths of the group of time series in the portfolio to which I would apply usable deterministic holding functions. The problem required mathematical skills that are beyond my present abilities. However, by bypassing the random component; I could simply put the problem aside for a future undertaking, giving me time to learn the necessary skills to manipulate stochastic data series and express my vision of the problem. I will still search for ways to modulate position sizing based on price movement, but for now, since I can't express in mathematical terms what I have in mind; I will have to be content to deal with the drift side of the holding equation. Thereby my current endeavor should be considered only as a subset of possible outcomes which could be improved upon with better market timing tools or better trading strategies.*

A direct consequence of by-passing the random component of the SDE equation is the removal of the noise - the error term - from the data series. What is left are linear equations - the regression lines of each stock composing the portfolio - having for average slope the portfolio's average rate of return which itself tends to the average market return (see Figure 1).

Figure 1: Stock Price Linear Regressions



Set of linear regressions lines representing stock prices.

Trading only on these linear regressions somehow does not change the game. It is still a stock picking process tailored by the underpinning of a Buy & Hold strategy.

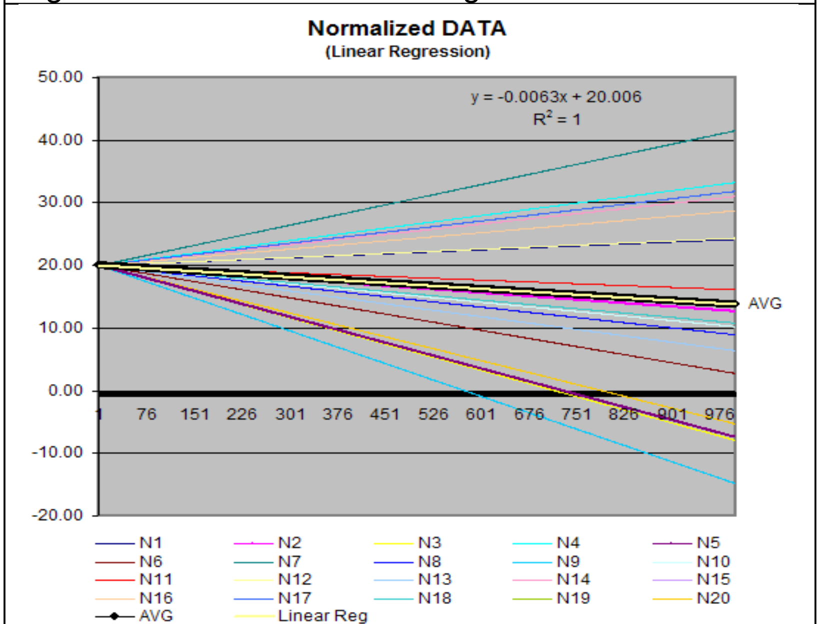
The stock prices could be normalized to a common starting price with their respective slopes representing their individual returns $\mu_j(t)$ (see Figure 2). Whatever the initial stock selection; having a long term investment horizon in mind, probabilities are that your own selection would also look somewhat like Figure 2.

Trading a Straight Line:

Based on Figure 2, one is confronted with a set of stock prices that vary linearly in time. Their respective terminal rate of return $\mu_j(t)$ can only be known at time *T*, and therefore forces us to re-include the SDE quasi-random evolutionary path of price movements. But what if, in the stock selection process itself, the stocks selected would have an average rate of return above the long term average? Then, what would the best solution be?

You need to trade a quasi-random $\mu_j(t)$ where all you can maybe estimate is the long term market average $\mu_M(t)$. You still don't know the order of each stock's ultimate return and therefore still don't know the future outcome of all your trading strategies or the final value of the portfolio; except in a general sense that your average return $\bar{\mu}_j(t)$ with tend to the future average market return $\bar{\mu}_M(t)$ as your trading horizon increases.

Figure 2: Normalized Linear Regressions



Normalizing the data in Figure 1 makes the distribution of random lines to fan out from the same origin. The rate of change in each price series is the same in both figures.

The separation of the SDE equation into its drift and random components should bring more insight to the portfolio problem. On the one hand, the quasi-random side of the equation having a zero long term expected value could have for underlying notion that any trading strategy whatsoever trying to exploit this component has zero for expected value as well. It's like trying to beat the heads or tails game which has been known for centuries to be unbeatable. There is no need to prove it again. You can win at the game but it will be purely by chance. This does not mean that if you play this stock market game it will end in zero profits, only that the expected value of the random component of the game is zero; the drift side has a positive bias. *This won't stop me from trying to find a tradable solution to the random component as I think that there are sufficient periods of "trendyness" in stock price movements to warrant the search; maybe by a predictable component or some form of trend following methodology.*

The drift side of the SDE equation represents linear return equations as depicted in Figure 2; and technically, if one profits from long term trading, it is from the average positive slope of these curves. And the notion of a Growth Optimal Portfolio (GOP) is probably derived from the acceptance that the average market return is the most likely outcome for your trading strategy and as such bears all the characteristics of the market averages.

The long term secular trend for market returns (including dividend reinvestment) has been about 10% per year over long time horizons (20 years or more). This amounts to an average $0.04 of drift per day or about $0.20 per week per $100 invested. Putting a million dollars on the table, one can expect an average appreciation of about $200 per week as compensation for all the risk taken. Excluding the random component for the reason of its zero expected value does not mean that it has been removed from the market. Historically, one standard deviation of annual returns is about 16%. On a monthly basis, this is a lot of risk for a very low expected return. Within one standard deviation, the market can easily move by 5 % in a month which represent some $50 000 on the million dollar portfolio; while at 2 standard deviations this can reach $100 000. This is very high volatility compared to expected reward.

But playing the straight line also implies that the Buy & Hold strategy is your best long term bet, on average. We seem to be coming full circle. You want to win a few extra percentage points investing in the market but have to take tremendous risk over extended periods of time to achieve this. And there is always a distinct possibility that unforeseen events (black swans) could wipe out 10 years of extra return in an instant. Therefore, there is an obligation to diversify and the more you diversify, the more your total return will approach the secular trend; again coming full circle. It appears as if you can only achieve what the market offers.

Trying to time the random component has its own pitfalls; its long term expected value is zero after all. Meaning that what ever strategy you devise; it is bound to work some of the time by pure chance alone. And over an extended period of time, all your trading efforts on the random component will tend to average out to a zero rate of return. Unless you can find some consistent and persistent price movement anomalies that could be slightly predictable (meaning finding an edge); you are at a great disadvantage at the game of generating some alpha. However, there is no way of extracting the random component from the game; and therefore, whatever positive expectancy trading system used trying to exploit the random component should nevertheless have returns that will approach the long term average market return minus trading costs. One seems to be rewarded with a GOP performance just by playing the game long enough!

Again, playing the straight line has for best strategy: the Buy & Hold. It does not matter which assets are being picked as long as they can appreciate in time. You can buy stocks, bonds, real estate even collectibles. The important point is to select assets that can be resold at a higher price or alternatively to short depreciating assets. The way to improve on the Buy & Hold strategy appears therefore to do more of the same by reinvesting the generated profits. This is what all my formulas amount to: reinvesting, over time, the excess equity buildup.

Trading the Buffett way:

This all leads to a Warren Buffett style of investing: select undervalued securities generating positive cash flow that will be there for a long time and make your bets big. Two things come to mind from his way of investing: 1) Buffett's most sought after holding period is naturally forever; 2) Buffett reinvests the generated profits. And to quote Buffett: *"The important thing is to keep playing, to play against weak opponents and to play for big stakes".*

In a way, my formulas simply confirm what Buffett has done all along.

$$WB_j(t) = h_0^j (1+ g^j)^{t-1} P_0^j (1+ r^j)^t \quad \text{for an investment in asset j}$$

$$WB(T) = \sum_{j=1,t=0}^{d,T} h_0^j (1+ g^j)^{t-1} P_0^j (1+ r^j)^t \quad \text{for the total portfolio over horizon T}$$

which expresses the delayed reinvestment of the excess equity buildup. This formula does not change what the price will be; there is no control possible over that, but it does describe how the inventory on hand should be managed. The improved result is not a simple Buy & Hold; it becomes an enhanced Buy & Hold as if on steroids. This is where the alpha can be generated. It is not only your ability to pick stocks with above average performance that counts; it’s your ability to continue to reinvest your profits in the same manner that generated the profits in the first place.